# Surface Circulation from Satellite Images: Reduced Model of the Black Sea

Etienne Huot, Isabelle Herlin, Giuseppe Papari, Karim Drifi

Etienne Huot[1,2], Isabelle Herlin[1], Giuseppe Papari[3], Karim Drifi[1]

[1]INRIA, Université de Paris-Est, CEREA
[2]Université de Versailles – Saint-Quentin-en-Yvelines
[3]Numerical rocks AS

**SURFACE CIRCULATION FROM SATELLITE IMAGES: REDUCED MODEL OF THE BLACK SEA**

**Abstract**
Estimating surface circulation from satellite images is a hot subject for a large range of applications. Motion estimation from image data has been studied for long in the literature of Image Processing, and more recently in that of Data Assimilation (DA). This paper describes how the construction of dedicated spaces for projecting motion and image fields allows applying DA methods with a reduced model and eases the estimation of surface circulation on the whole Black Sea basin.

**Keywords**
Reduced model, waveform, data assimilation, image processing

## 1. Introduction

Estimating surface circulation from satellite images is a hot subject for a large range of applications. Littoral and oil spill monitoring are well-known issues for which precise nowcasting from image data has a high economical and social value. Motion estimation from image data has been studied for long in the literature of Image Processing [1,2,3,4], and more recently in that of Data Assimilation (DA) [5,6,7]. This paper describes how the construction of dedicated spaces for projecting motion and image fields allows applying DA methods with a reduced model and eases the estimation of surface circulation on the whole Black Sea basin. Galerkin projection of a model defined at the pixel level has already been used to estimate motion from images with bases obtained by Principal Order Decomposition (POD) [8,9]. The two major drawbacks of these methods are: 1- the difficulty to determine the reduced motion space. 2- Applying POD on the available image data means that the reduced model has to be recomputed for each new sequence, which increases computational time. In this paper, reduced spaces are spanned by bases that are automatically determined from the mathematical description of image and motion fields [10]. As these bases are once determined for quite, the reduced model is usable for each new satellite sequence.

The paper is organized as follows. Section 2 summarizes the data assimilation approach. Section 3 describes the motion and image bases dedicated to Black Sea. The Galerkin projection of the full model is explained in Section 4. Section 5 describes the data that have been used to validate and quantify the approach.

## 2. Data assimilation

Let $\mathbf{X}$ be the state vector depending of position $\mathbf{x}$ and date $t$ on a space-time domain $A = \Omega \times [0, T]$. The objective is to retrieve $X$ on the whole domain $A$ from observation values. $X$ is supposed to satisfy the following evolution equation on $[0, T]$:

$$\frac{\partial \mathbf{X}}{\partial t}(\mathbf{x}, t) + \mathbb{M}(\mathbf{X})(\mathbf{x}, t) = 0, \tag{1}$$

Let $\mathbf{Y}(\mathbf{x}, t)$ the observation value acquired at position $\mathbf{x}$ and date $t$. It is linked to the state vector by the observation equation:

$$\mathbf{Y}(\mathbf{x}, t) = \mathbb{H}(\mathbf{X})(\mathbf{x}, t) + \mathcal{E}_o(\mathbf{x}, t) \tag{2}$$

$\mathbb{H}$ is the observation operator and $\mathcal{E}_o$ the observation error.

An approximative knowledge, $\mathbf{X}_b(\mathbf{x})$, of the initial value $\mathbf{X}(\mathbf{x}, 0)$ is supposed to be available, that provides the background equation:

$$\mathbf{X}(\mathbf{x}, 0) = \mathbf{X}_b(\mathbf{x}) + \mathcal{E}_b(\mathbf{x}) \tag{3}$$

with $\mathcal{E}_b(\mathbf{x})$ the background error. The system to be solved for estimating $\mathbf{X}(\mathbf{x}, t)$ is composed of Eqs. (1), (2) and (3). The variables $\mathcal{E}_o(\mathbf{x}, t)$ and $\mathcal{E}_b(\mathbf{x})$ are supposed to be Gaussian, zero-mean and not correlated.

A cost function is designed, whose minimization provides the solution $\mathbf{X}(0)$:

$$\begin{aligned} J[\mathbf{X}(0)] \quad &= \int_{\Omega} (\mathbf{X}(\mathbf{x}, 0) - \mathbf{X}_b(\mathbf{x}))^T B^{-1}(\mathbf{x})(\mathbf{X}(\mathbf{x}, 0) - \mathbf{X}_b(\mathbf{x})) d\mathbf{x} \\ &+ \int_{A} [\mathbb{H}\,\mathbf{X} - \mathbf{Y}]^T(\mathbf{x}, t) R^{-1}(\mathbf{x}, t)[\mathbb{H}\mathbf{X} - \mathbf{Y}](\mathbf{x}, t) d\mathbf{x} dt \end{aligned} \tag{4}$$

## 3. Motion and image bases for Black Sea

Let $BS$ denote the Black Sea domain and $\mathbf{x}$ one pixel of $BS$. $\mathbf{w}(\mathbf{x}, t)$ is the velocity at pixel $\mathbf{x}$ and date $t$, $\mathbf{w}(t)$ the motion field at date $t$, and $\mathbf{w}$ the motion function on the space-time domain belonging to the Hilbert functional space $\mathfrak{F}$. The image function $I$ belongs to the Hilbert functional space $\mathfrak{I}$. Motion fields and images are considered to take values in vectorial subspaces spanned by the orthogonal bases $\Phi = \{\phi_i(\mathbf{x})\}_{i=1\ldots K}$ and $\Psi = \{\psi_j(\mathbf{x})\}_{j=1\ldots L}$. As explained in [10], it is possible to automatically derive the bases $\Phi$ and $\Psi$ from their mathematical properties. Surface motion $\mathbf{w}$ is then approximated as a divergence-free motion field and the basis $\Phi$ is searched in the vectorial space of divergence-free motion. Moreover, the motion fields $\mathbf{w}(t)$ are assumed to be smooth and the same constraint is applied to the basis elements. The normal motion vector is considered as null value on the boundary of the Black Sea domain and Dirichlet boundary conditions are applied to the basis elements. Consequently, the basis elements $\phi_1, \ldots, \phi_K$ are obtained as the $K$ first solutions of the following minimization problem:

$$\begin{cases} \phi_1 = \arg \min\limits_{\phi \in \mathfrak{F}_{\mathcal{B}}} \mathcal{Q}(\phi), & |\phi|^2 = 1 \\ \phi_2 = \arg \min\limits_{\phi \in \mathfrak{F}_{\mathcal{B}}} \mathcal{Q}(\phi), & |\phi|^2 = 1, \phi \perp \phi_1 \\ \quad \cdots & \\ \phi_k = \arg \min\limits_{\phi \in \mathfrak{F}_{\mathcal{B}}} \mathcal{Q}(\phi), & |\phi|^2 = 1, \phi \perp \phi_j, \forall i < k \\ \quad \cdots & \end{cases} . \tag{5}$$

where:

$$\mathcal{Q}(\phi) \triangleq \int_{\Omega} |\nabla \phi(\mathbf{x})|^2 \mathrm{d}\mathbf{x}$$

and $\mathfrak{F}_{\mathcal{B}}$ denotes the subspace of $\mathfrak{F}$ of divergence-free motion with null normal value on the boundary of $BS$. Some $\phi_i$ elements obtained from that minimization issue are given in Figure 1 for $i = 1, 4, 128$ and $512$. Scales modeled by these basis elements are finer as $i$ value is increasing. The image basis $\Psi$ is obtained in the same way.

Figure 1 Elements 1, 4, 128 and 512 of the divergence-free motion basis

## 4. Model reduction

Let consider the following assumptions on the dynamics displayed by image sequences: motion is constant along pixels' trajectories and pixels' image values are transported by motion:

$$\begin{cases} \dfrac{\partial \mathbf{w}}{\partial t}(\mathbf{x}, t) + (\mathbf{w} \cdot \nabla)\mathbf{w}(\mathbf{x}, t) & = & 0 \\ \dfrac{\partial I}{\partial t}(\mathbf{x}, t) + \mathbf{w} \cdot \nabla I(\mathbf{x}, t) & = & 0 \end{cases} . \tag{6}$$

When projecting motion fields and images on the vectorial spaces spanned by $\Phi$ and $\Psi$, it comes:

$$\begin{aligned} \mathbf{w}(\mathbf{x},t) &\approx \sum_{i=1}^{K} a_i(t)\phi_i(\mathbf{x}) \\ I(\mathbf{x},t) &\approx \sum_{j=1}^{L} b_j(t)\psi_j(\mathbf{x}) \end{aligned} \tag{7}$$

When replacing in System (6), we obtain the following ordinary differential equations for the projection coefficients:

$$\begin{aligned} \frac{da_k}{dt} + \sum_{i,j=1}^{K,K} a_i a_j \frac{\langle(\phi_i \cdot \nabla)\phi_j, \phi_k\rangle}{\langle\phi_k, \phi_k\rangle} = 0 \\ \frac{db_l}{dt} + \sum_{i,j=1}^{K,L} a_i b_j \frac{\langle(\phi_i \cdot \nabla)\psi_j, \psi_l\rangle}{\langle\psi_l, \psi_l\rangle} = 0 \end{aligned} \tag{8}$$

We define $a(t) = (a_1(t)\ldots a_K(t))^T$ and $b(t) = (b_1(t)\ldots b_L(t))^T$.

A reduced state vector is defined as $\mathbf{X}_R(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$ and System (8) is summarized by:

$$\frac{d\mathbf{X}_R}{dt}(t) + \mathbb{M}_R(\mathbf{X}_R(t)) = 0 \tag{9}$$

the reduced model $\mathbb{M}_R$ being the Galerkin projection of the full model on bases $\Phi$ and $\Psi$.

## 5. Results

A twin experiment has been designed to test the approach. Given an analysis result, on 2011 July 8th, of the sea surface height anomaly from the MyOcean project website[1], the corresponding geostrophic motion field is computed and displayed on the left part of Figure 2. The corresponding SST image is displayed on the right.

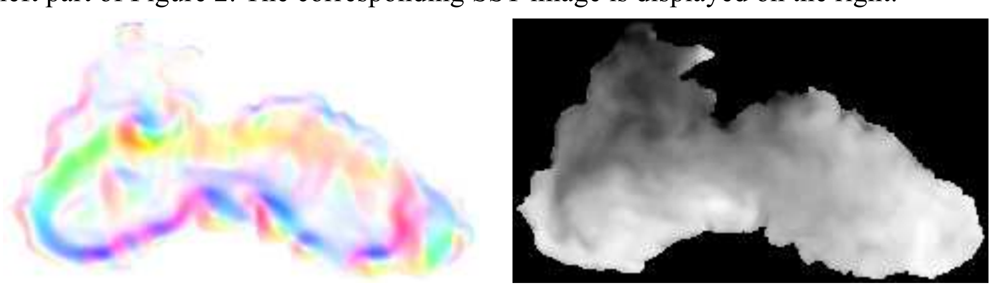
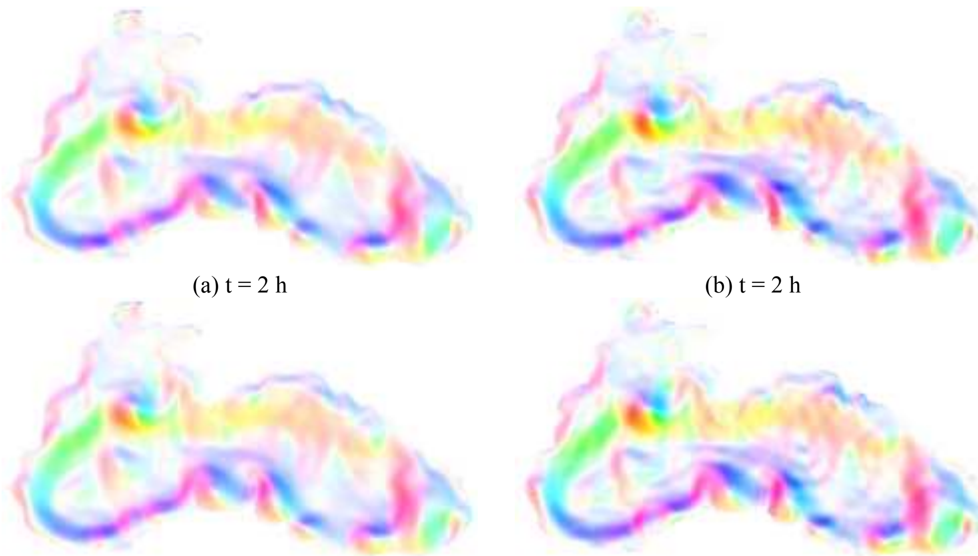


Figure 2 Initial conditions of the simulation – $\mathbf{w_0}$ (Left) and $I_0$ (Right).

[1] http://www.myocean.eu

These fields are used as initial conditions of System (6) in order to obtain ground-truth data on motion and images and being able to quantify the approach.

Snapshots $I^{obs}(t_k)$ of the simulated image sequence are projected on the vectorial space spanned by $\Psi$ in order to get the $b^{obs}(t_k)$ coefficients vectors used for assimilation in the reduced model. The result on motion is displayed on the right of Figure 3 and compared to the ground-truth on the left.



(a) t = 2 h (b) t = 2 h

(c) t = 50 h (d) t = 50 h

Figure 3 Ground truth (Left), motion estimated by assimilation (Right).

The NRMSE on the motion norm is less than 5% and the angular error mean is less that 2 degrees on the whole temporal interval.

## 6. Conclusion

The paper describes a process for estimating surface circulation from satellite images. It is based on an image assimilation method in a reduced model. That last is obtained by the Galerkin projection of surface circulation equations on bases that have been designed for Black Sea.

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